

Grade 7 Mathematics Item Specification C1 TA

Claim 1: Concepts and Procedures

Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Content Domain: Ratios and Proportional Relationships

Target A [m]: Analyze proportional relationships and use them to solve real-world and mathematical problems. (DOK 2)

Tasks for this target will require students to identify and represent proportional relationships in various formats (tables, graphs, equations, diagrams, verbal descriptions) and interpret specific values in context. (See 7.G Target E for possible context.) Other tasks will require students to compute unit rates including those associated with ratios of fractions.

<p>Standards: 7.RP.A, 7.RP.A.1, 7.RP.A.2, 7.RP.A.3</p>	<p>7.RP.A Analyze proportional relationships and use them to solve real-world and mathematical problems.</p> <p>7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. <i>For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{1/2}{1/4}$ miles per hour, equivalently 2 miles per hour.</i></p> <p>7.RP.A.2 Recognize and represent proportional relationships between quantities.</p> <ol style="list-style-type: none"> Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. Represent proportional relationships by equations. <i>For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.</i> Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate. <p>7.RP.A.3 Use proportional relationships to solve multi-step ratio and percent problems. <i>Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.</i></p>
<p>Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling:</p> <p>6.RP.A, 6.RP.A.1, 6.RP.A.2, 6.RP.A.3</p> <p>8.EE.B, 8.EE.B.5</p>	<p>Related Grade 6 Standards</p> <p>6.RP.A Understand ratio concepts and use ratio reasoning to solve problems.</p> <p>6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. <i>For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."</i></p> <p>6.RP.A.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. <i>For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $\frac{3}{4}$ cup of flour for each cup of sugar." "We paid \$75 for 15 hamburgers, which is a rate of \$5 per</i></p>

	<p><i>hamburger.</i>"</p> <p>6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</p> <ol style="list-style-type: none"> Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. Solve unit rate problems including those involving unit pricing and constant speed. <i>For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?</i> Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. <p>Related Grade 8 Standards</p> <p>8.EE.B Reason about and solve one-variable equations and inequalities.</p> <p>8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. <i>For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.</i></p>
DOK Level:	2
Achievement Level Descriptors:	
<p>RANGE Achievement Level Descriptor (Range ALD) Target A: Analyze proportional relationships and use them to solve real-world and mathematical problems.</p>	Level 1 Students should be able to identify proportional relationships presented in graphical, tabular, or verbal formats in familiar contexts.
	Level 2 Students should be able to find whole number proportionality constants in relationships presented in graphical, tabular, or verbal formats in familiar contexts. They should also be able to identify proportional relationships presented in equation formats and find unit rates involving whole numbers.
	Level 3 Students should be able to identify, represent, and analyze proportional relationships in various formats; find unit rates associated with ratios of fractions; and use unit rates to solve one-step problems involving rational numbers. They should be able to analyze a graph of a proportional relationship in order to explain what the points (x, y) and $(1, r)$ represent, where r is the unit rate, and use this information to solve problems.
	Level 4 Students should be able to solve real-world problems involving proportional relationships and measurement conversions in various formats (e.g., verbally, tabular, graphically) in a contextual scenario that involves identifying relationships between elements presented in various formats.
Evidence Required:	1. The student computes unit rates and finds the constant of proportionality of proportional relationships in various forms.

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	<p>2. The student determines whether two quantities, shown in various forms, are in a proportional relationship.</p> <p>3. The student represents proportional relationships between quantities using equations.</p> <p>4. The student interprets specific values from a proportional relationship in the context of a problem situation.</p> <p>5. The student computes with percentages in context.</p>
Allowable Response Types:	Equation/Numeric; Multiple Choice, multiple correct response; Matching Tables
Allowable Stimulus Materials:	two-way tables, graphs of both nonlinear and linear equations, equations in two variables, diagrams
Construct-Relevant Vocabulary:	proportional relationship, ratio, unit rate, constant of proportionality, origin, percent increase, percent decrease, percent error
Allowable Tools:	Calculator
Target-Specific Attributes:	
Non-Targeted Constructs:	
Accessibility Guidance:	<p>Item writers should consider the following Language and Visual Element/Design guidelines¹ when developing items.</p> <p>Language Key Considerations:</p> <ul style="list-style-type: none"> • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references <p>Visual Elements/Design Key Considerations:</p> <ul style="list-style-type: none"> • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics <p>Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.²</p>
Development Notes:	The graphical representations of real-world situations are assessed in Claims 2 and 3.

¹ For more information, refer to the General Accessibility Guidelines at:

<http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf>

² For more information about student accessibility resources and policies, refer to

http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

<p>Task Model 1</p> <p>Response Type: Equation/Numeric</p> <p>DOK Level 2</p> <p>7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.</p> <p>7.RP.A.2b Recognize and represent proportional relationships between quantities. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.</p> <p>Evidence Required: 1. The student computes unit rates and finds the constant of proportionality of proportional relationships in various forms.</p> <p>Tools: Calculator</p>	<p>Prompt Features: The student is prompted to give the constant of proportionality between two quantities in a proportional relationship.</p> <p>Stimulus Guidelines:</p> <ul style="list-style-type: none"> • Ratios in the proportional relationship should be ratios of fractions. • Context should be familiar to students 12 to 14 years old. • Item difficulty can be adjusted via these example methods: <ul style="list-style-type: none"> ○ Fractions can be expressed as mixed numbers or not. ○ Constants of proportionality can be whole numbers or fractions. <p>TM1a Stimulus: The student is presented with a verbal description of a real-world situation involving a proportional relationship.</p> <p>Example Stem: David uses $\frac{1}{4}$ cup of apple juice for every $\frac{1}{2}$ cup of carrot juice to make a fruit drink.</p> <p>Enter the number of cups of apple juice David uses for 1 cup of carrot juice.</p> <p>Rubric: (1 point) The student enters the correct number (e.g., $\frac{1}{2}$).</p> <p>Response Type: Equation/Numeric</p> <p>TM1b Stimulus: The student is presented with a table or diagram of a proportional relationship in a context.</p> <p>Example Stem 1: This table shows a proportional relationship between the number of cups of sugar and flour used for a recipe.</p> <table border="1" data-bbox="664 1247 1230 1381"> <thead> <tr> <th>Cups of Sugar</th><th>Cups of Flour</th></tr> </thead> <tbody> <tr> <td>2</td><td>5</td></tr> <tr> <td>6</td><td>15</td></tr> <tr> <td>8</td><td>20</td></tr> </tbody> </table> <p>Enter the number of cups of sugar used for 1 cup of flour.</p> <p>Example Stem 2: This table shows a proportional relationship between the number of cups of sugar and flour used for a recipe.</p> <table border="1" data-bbox="664 1575 1230 1738"> <thead> <tr> <th>Cups of Sugar</th><th>Cups of Flour</th></tr> </thead> <tbody> <tr> <td>$2\frac{1}{2}$</td><td>$7\frac{1}{2}$</td></tr> <tr> <td>$3\frac{3}{4}$</td><td>$11\frac{1}{4}$</td></tr> </tbody> </table> <p>Enter the number of cups of sugar used for 1 cup of flour.</p> <p>Rubric: (1 point) Student enters the correct number (e.g., $\frac{2}{5}$; $\frac{1}{3}$).</p> <p>Response Type: Equation/Numeric</p>	Cups of Sugar	Cups of Flour	2	5	6	15	8	20	Cups of Sugar	Cups of Flour	$2\frac{1}{2}$	$7\frac{1}{2}$	$3\frac{3}{4}$	$11\frac{1}{4}$
Cups of Sugar	Cups of Flour														
2	5														
6	15														
8	20														
Cups of Sugar	Cups of Flour														
$2\frac{1}{2}$	$7\frac{1}{2}$														
$3\frac{3}{4}$	$11\frac{1}{4}$														

<p>Task Model 1</p> <p>Response Type: Equation/Numeric</p> <p>DOK Level 2</p> <p>7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.</p> <p>7.RP.A.2b Recognize and represent proportional relationships between quantities. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.</p> <p>Evidence Required: 1. The student computes unit rates and finds the constant of proportionality of proportional relationships in various forms.</p> <p>Tools: Calculator</p> <p>Version 3 Update: Retired TM1c and revised TM1d example stems.</p>	<p>Prompt Features: The student is prompted to give the constant of proportionality for a proportional relationship between two quantities.</p> <p>Stimulus Guidelines:</p> <ul style="list-style-type: none"> Context should be familiar to students 12 to 14 years old. Item difficulty can be adjusted via these example methods: <ul style="list-style-type: none"> The equation should come in the following forms: $y = rx$, where r is the constant of proportionality and $[\text{coefficient1}][\text{variable1}] = [\text{coefficient2}][\text{variable2}]$. The constant of proportionality can be a whole number, positive fraction, or mixed number. Coefficients include whole numbers, fractions, and exclude the number one. <p>TM1d Stimulus: The student is presented with an equation of a proportional relationship.</p> <p>Example Stem 1: A drink recipe calls for papaya juice and carrot juice. This equation represents the proportional relationship between the number of quarts of papaya juice (p) and carrot juice (c) in the recipe.</p> $2p = 8c$ <p>Enter the number of quarts of papaya juice used for 1 quart of carrot juice.</p> <p>Example Stem 2: A drink recipe calls for papaya juice and carrot juice. This equation represents the proportional relationship between the number of quarts of papaya juice (p) and carrot juice (c) in the recipe.</p> $(1\frac{1}{3})p = (3\frac{1}{3})c$ <p>Enter the number of quarts of papaya juice used for 1 quart of carrot juice.</p> <p>Rubric: (1 point) The student enters the correct number (e.g., $4; \frac{5}{2}$).</p> <p>Response Type: Equation/Numeric</p>
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Task Model 2

Response Type:
Multiple Choice,
multiple correct
response

DOK Level 2

7.RP.A.2a

Recognize and represent proportional relationships between quantities.
a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Evidence Required:

2. The student determines whether two quantities, shown in various forms, are in a proportional relationship.

Tools: Calculator

Prompt Features: The student is prompted to identify tables of values that represent proportional relationships.

Stimulus Guidelines:

- Tables should be labeled and have four to five ordered pairs. All tables within an item should follow the same format.
- Where possible, tables should contain values arising out of contextual relationships.
- Item difficulty can be adjusted via these example methods:
 - Table values are whole numbers or fractions.
 - Fractions may be mixed numbers.
 - For graphs, distractors should include graphs with the equation in the form of $y = x^2$ and the equation in the form of $y = mx + b$ (where $b \neq 0$).

TM2a

Stimulus: The student is presented with one table per answer choice. Where possible, include a contextual reason for the tables of relationships.

Example Stem 1: Select **all** tables that represent a proportional relationship between x and y .

A.

x	0	1	2	3
y	0	2	4	6

B.

x	0	2	4	6
y	0	4	16	36

C.

x	0	3	6	9
y	0	15	30	45

D.

x	0	4	6	8
y	0	16	36	64

Answer Choices: Answer choices should be tables showing a relationship between two quantities. There should be one to two tables showing proportional relationships. Distractors should be tables that do not show a proportional relationship, which may include a relationship following an equation in the form of $y = mx + b$ (where $b \neq 0$) or $y = x^2$.

Rubric: (1 point) Student selects all the correct tables. (e.g., A and C).

Response Type: Multiple Choice, multiple correct response

Task Model 2

Response Type:
Multiple Choice,
multiple correct
response

DOK Level 2**7.RP.A.2a**

Recognize and represent proportional relationships between quantities.
a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Evidence Required:

2. The student determines whether two quantities, shown in various forms, are in a proportional relationship.

Tools: Calculator

Example Stem 2: Select **all** tables that represent a proportional relationship between x and y .

A.

x	0	1	2	3
y	0	2	4	6

B.

x	0	2	4	6
y	0	4	16	36

C.

x	0	$\frac{1}{9}$	$\frac{1}{4}$	$\frac{1}{2}$
y	0	$\frac{1}{81}$	$\frac{1}{16}$	$\frac{1}{4}$

D.

x	0	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
y	0	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$

Answer Choices: Answer choices should be tables showing a relationship between two quantities. There should be one to two tables showing proportional relationships. Distractors should be tables that do not show a proportional relationship, which may include a relationship following an equation in the form of $y = mx + b$ (where $b \neq 0$) or $y = x^2$.

Rubric: (1 point) Student selects all the correct tables. (e.g., A and D).

Response Type: Multiple Choice, multiple correct response

Task Model 2

Response Type:
Multiple Choice,
multiple correct
response

DOK Level 2**7.RP.A.2a**

Recognize and represent proportional relationships between quantities.
a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Evidence Required:

2. The student determines whether two quantities, shown in various forms, are in a proportional relationship.

Tools: Calculator

Prompt Features: The student is prompted to identify which graphs represent proportional relationships.

Stimulus Guidelines:

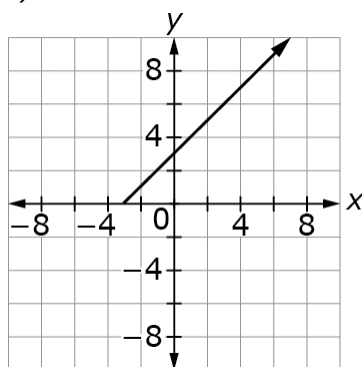
- Context should be familiar to students 12 to 14 years old.
- Item difficulty can be adjusted via these example methods:
 - Unit rate is a whole number or fraction.
 - Distractors should include graphs with the equation in the form of $y = x^2$ and the equation in the form of $y = mx + b$ (where $b \neq 0$).

TM2b

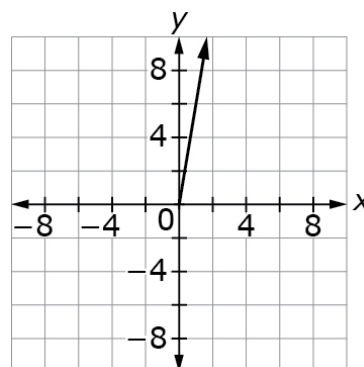
Stimulus: The student is presented with one table or one graph per answer choice.

Example Stem: Select **all** the graphs that represent a proportional relationship between x and y .

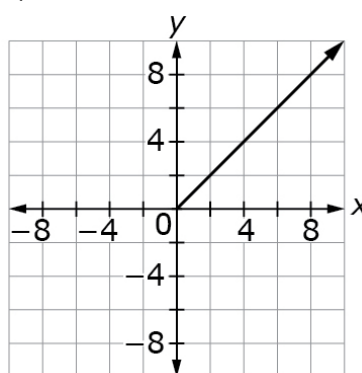
A)



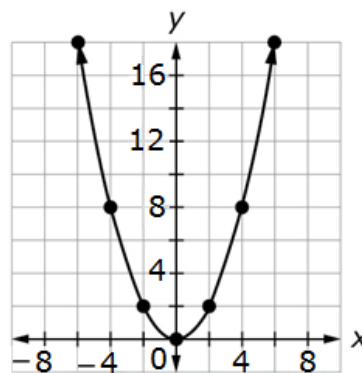
C)



B)



D)



Answer Choices: Distractors should be graphs that do not show a proportional relationship, which may show a nonlinear relationship or a relationship following an equation in the form of $y = mx + b$ (where $b \neq 0$) or $y = x^2$.

Rubric: (1 point) Student selects all the correct graphs (e.g., B and C).

Response Type: Multiple Choice, multiple correct response

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Task Model 3

Response Type:
Equation/Numeric

DOK Level 2

7.RP.A.2c

Recognize and represent proportional relationships between quantities.
c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t = pn$.*

Evidence Required:

3. The student represents proportional relationships between quantities using equations.

Tools: Calculator

Prompt Features: The student is prompted to give an equation that represents the proportional relationship between two given quantities.

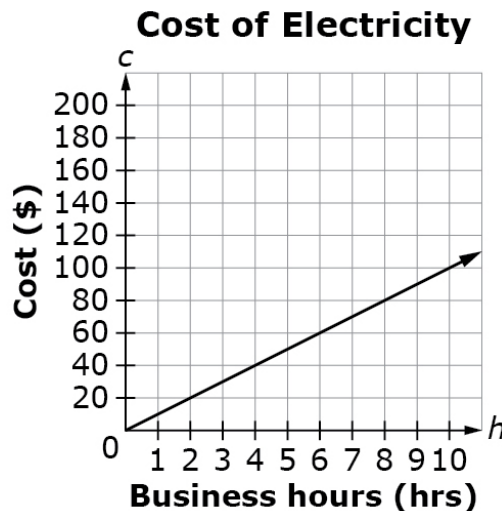
Stimulus Guidelines:

- Context should be familiar to students 12 to 14 years old.
- Graph is linear and begins at (0, 0) or a set of plotted points which includes (0, 0).
- Tables should be labeled, represent the relationship between two variables, and have 3-5 ordered pairs.
- For graphs, axes are labeled and include whole numbers and/or fractions.
- The constant of proportionality is a whole number or fraction.
- Item difficulty can be adjusted via these example methods:
 - Scaling of the graph may be fractional or in units other than multiples of 2 or 10.
 - Table values are whole numbers or fractions.
 - Fractions are not mixed numbers.

TM3

Stimulus: The student is presented with two quantities in a contextual proportional relationship given in a graph or table.

Example Stem 1: This graph shows the relationship between the number of hours (h) a business operates and the total cost of electricity (c).



Find the constant of proportionality (r) for this relationship. Using the value for k , enter an equation in the form of $c = rh$ that represents the relationship between the number of hours (h) and the total cost (c).

Task Model 3

Response Type:
Equation/Numeric

DOK Level 2

7.RP.A.2c

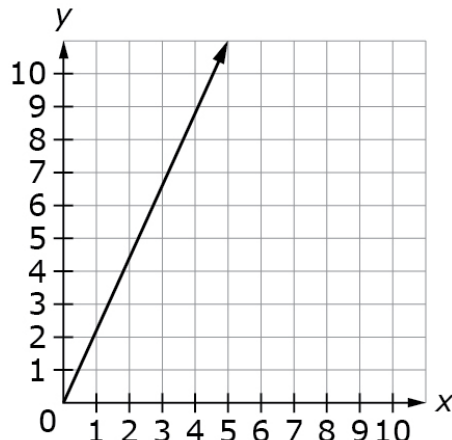
Recognize and represent proportional relationships between quantities.
c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t = pn$.*

Evidence Required:

3. The student represents proportional relationships between quantities using equations.

Tools: Calculator

Example Stem 2: This graph shows a proportional relationship between x and y .



Find the constant of proportionality (k). Using the value for k , enter an equation in the form of $y = kx$.

Example Stem 3: This table shows a proportional relationship between x and y .

x	y
4	48
5	60
8	96

Find the constant of proportionality (k). Using the value for k , enter an equation in the form of $y = kx$.

Rubric: (1 point) Student enters the correct equation (e.g., $c = 10h$; $y = 2x$; $y = 12x$).

Response Type: Equation/Numeric

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Task Model 4

Response Type: Matching Tables

DOK Level 2

7.RP.A.2d

Recognize and represent proportional relationships between quantities.
d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.

Evidence Required:

4. The student interprets specific values from a proportional relationship in the context of a problem situation.

Tools: Calculator

Prompt Features: The student is prompted to select specific values from a proportional relationship in the context of a problem situation.

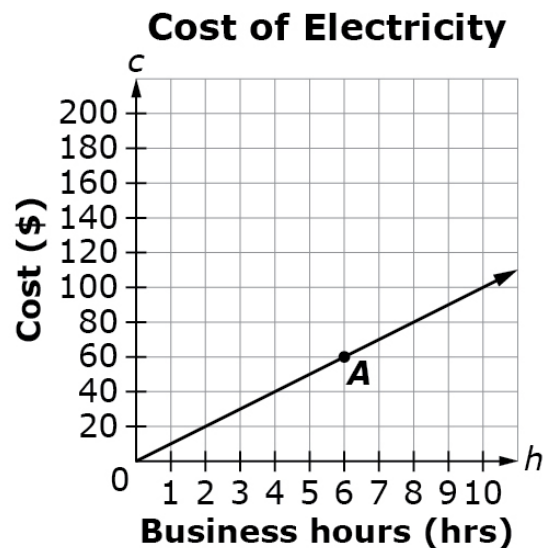
Stimulus Guidelines:

- Context should be familiar to students 12 to 14 years old.
- Graph is linear and begins at $(0, 0)$ or a set of plotted points which includes $(0, 0)$.
- Graph axes are labeled and include whole numbers and/or fractions.
- The constant of proportionality is a whole number or fraction.
- Item difficulty can be adjusted via these example methods:
 - One answer choice which assesses the interpretation of a single point on the graph that is not the unit rate is easier than an answer choice that compares the interpretation of two different points.

TM4

Stimulus: The student is presented with a graph of a proportional relationship where specific values may be emphasized.

Example Stem: This graph shows the relationship between the number of hours (h) a business operates and the total cost (c) of electricity.



Select True or False for each statement about the graph.

Statement	True	False
Point A represents the total cost of electricity when operating the business for 6 hours.		
The total cost of electricity is \$8 when operating the business for 80 hours.		
The total cost of electricity is \$10 when operating the business for 1 hour.		

	<p>Rubric: (1 point) Student determines each statement as being either true or false (e.g., T, F, T). Each statement is a sentence describing one of the points in the context. False statements should be statements that use the wrong values or switch the values when interpreting the graph. More difficult statements are about points beyond the visible portion of the graph.</p> <p>Response Type: Matching Tables</p>
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<p>Task Model 5</p> <p>Response Type: Equation/Numeric</p> <p>DOK Level 2</p> <p>7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. <i>Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.</i></p> <p>Evidence Required: 5. The student computes with percentages in context.</p> <p>Tools: Calculator</p>	<p>Prompt Features: The student is prompted to compute with percentages in a real-world context that requires multiple steps to solve.</p> <p>Stimulus Guidelines:</p> <ul style="list-style-type: none"> Context of problems should be familiar to students 12 to 14 years old. For items asking for a percentage, the percent symbol (%) should not be required for full credit. For items asking for a dollar amount, the dollar sign (\$) should not be required for full credit. Item difficulty can be adjusted via these example methods: <ul style="list-style-type: none"> 1-3 step(s) problem. Multiplying by a percent which should include benchmark percentages, i.e., 25%, 50%, etc. Divide two numbers or by a percent which should include benchmark percents, 25%, 50%, etc. <p>TM5</p> <p>Stimulus: The student is presented with a real-world context involving adding or subtracting a percent to the whole (simple interest, tax, commission, markup, markdowns, tips, coupons, and discounts).</p> <p>Example Stem 1: Dave buys a baseball for \$15 plus an 8% tax. Mel buys a football for \$20 plus an 8% tax. Enter the difference in the amount Dave and Mel paid, including tax. Round your answer to the nearest cent.</p> <p>Rubric: (1 point) Student gives the correct difference in the amount between David and Mel (e.g., 5.40).</p> <p>Response Type: Equation/Numeric</p> <p>Example Stem 2: A bicycle is originally priced at \$80. The store owner gives a discount and the bicycle is now priced at \$60. Enter the percentage discount for the cost of the bicycle.</p> <p>Rubric: (1 point) Student gives the correct percentage discount (e.g., 25).</p> <p>Response Type: Equation/Numeric</p> <p>Example Stem 3: Dave has a 32 ounce energy drink. He drinks 10 ounces. Enter the percentage of ounces Dave has left of his energy drink. Round your answer to the nearest hundredth.</p> <p>Rubric: (1 point) Student gives the correct percentage (e.g., 68.75).</p> <p>Response Type: Equation/Numeric</p>
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